Name: Key

Block: X

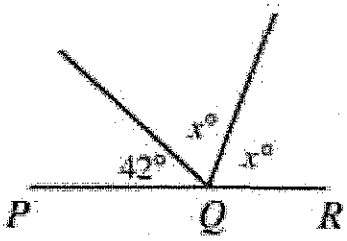
Problem Solving Quiz 2

This quiz is in two sections. For each section, you must select ***FOUR*** of the six available problems to complete. Please ***draw an asterix or circle the questions*** that you wish to be assessed! You are welcome to attempt all questions, but only 4 from each will be assessed.

For each selected question, do your best to reach a correct answer and ***provide justification for that answer***. Your justification may include diagrams, mathematical work, written explanations, graphs, or other methods. You are encouraged to ***try to justify your answers in multiple ways.*** *You will be assessed on both your answer and your reasoning and justification.*

Section 1

1. Points P, Q, R lie in a straight line. The value of x is



∠s on a straight line add to 180°, so

$42^\circ + x + x = 180^\circ$

$42^\circ + 2x = 180^\circ$

$-42^\circ \qquad -42^\circ$

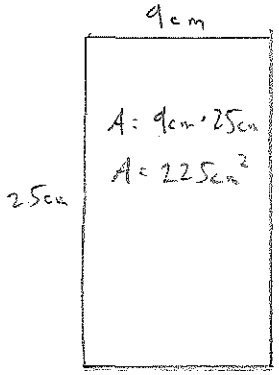
$2x = 138^\circ$

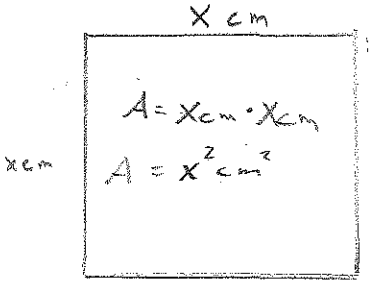
$\div 2 \qquad \div 2$

$x = 69^\circ$

The value of x is 69°

2. A rectangular sheet of paper measures 25 cm by 9 cm. The dimensions of a square sheet of paper with the same area are





The rectangle's area is equal to the square's, so

$225\,cm^2 = x^2\,cm^2$

$225 = x^2$

$\sqrt{} \qquad \sqrt{}$

$15 = x$

The square has dimensions 15cm x 15cm.

3. The number 519 is formed using the digits 5, 1 and 9. The three digits of this number are rearranged to form the largest possible and then the smallest possible three digit numbers. What is the difference between these largest and smallest numbers?

Largest is 951. Because $100 > 10 > 1$, we want to order our digits greatest to least. By a similar line of reasoning, 159 is the smallest.

largest − smallest = difference

951 − 159 = difference

792 = difference

4. The value of $(2+3)^2 - (2^2+3^2)$ is

B
E
D
M
A
S

$(2+3)^2 - (2^2+3^2)$

$= (5)^2 - (4+9)$

$= 25 - (13)$

$= 12$

5. The ratio of the value of four nickels (5¢) to the value of six dimes (10¢) to the value of two quarters (25¢) can be written as

(A) 4 : 6 : 2 (B) 2 : 6 : 5 (C) 2 : 3 : 1 (D) 6 : 4 : 2 (E) 1 : 2 : 3

Value 4 nickels = $4 \cdot 5¢ = 20¢$

Value 6 dimes = $6 \cdot 10¢ = 60¢$

Value 2 quarters = $2 \cdot 25¢ = 50¢$

$V_n : V_d : V_q = 20¢ : 60¢ : 50¢$

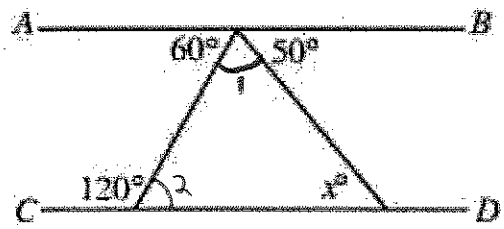
$= 20 : 60 : 50$ (remove units)

$\div 10 \quad \div 10 \quad \div 10$

$= 2 : 6 : 5$

∴ B

6. In the diagram, AB and CD are straight lines. The value of x is



$\angle 1$: angles on a straight line add to 180°, so $60° + \angle 1 + 50° = 180°$

$\angle 1 + 110° = 180°$

$\angle 1 = 70°$

$\angle 2$: angles on a line add to 180°, so

$120° + \angle 2 = 180°$

$\angle 2 = 60°$

$\angle X$: $\angle$s in $\triangle$ add to 180°, so

$\angle 1 + \angle 2 + \angle X = 180°$

$70° + 60° + x° = 180°$

$130° + x° = 180$

$x° = 50°$

The value of x is 50

OR: ① show $AB \parallel CD$, the Alt $\angle$s are =.

Section 2

1. In which set of scores is the median greater than the mean?

(A) 10, 20, 40, 40, 40 (B) 40, 50, 60, 70, 80 (C) 20, 20, 20, 50, 80
(D) 10, 20, 30, 100, 200 (E) 50, 50, 50, 50, 100

Since they are listed least to greatest, the median is the middle value for each set

We compute each mean: mean: $\frac{\text{sum of \#s}}{\text{no. of \#s}}$

	Mean	Median
A:	30	40
B:	60	60
C:	38	20
D:	72	30
E:	60	50

only set A meets our criteria.

2. Which of these values is the largest?

(A) $\frac{4}{2-\frac{1}{4}}$ (B) $\frac{4}{2+\frac{1}{4}}$ (C) $\frac{4}{2-\frac{1}{3}}$ (D) $\frac{4}{2+\frac{1}{3}}$ (E) $\frac{4}{2-\frac{1}{2}}$

(1) Evaluate each

$= \frac{4}{\left(\frac{7}{4}\right)}$ $= \frac{4}{\left(\frac{9}{4}\right)}$ $\frac{4}{\left(\frac{5}{3}\right)}$ $\frac{4}{\left(\frac{7}{3}\right)}$ $\frac{4}{\left(\frac{3}{2}\right)}$

$= \frac{16}{7}$ $= \frac{16}{9}$ $= \frac{12}{5}$ $= \frac{12}{7}$ $= \frac{8}{3}$

$= 2\frac{2}{7}$ $= 1\frac{7}{9}$ $= 2\frac{2}{5}$ $= 1\frac{5}{7}$ $= 2\frac{2}{3}$

Thirds > Fifths > Sevenths, so E. OR: Smaller denominator → larger value.

3. A student may pay \$1.50 for a single bus ticket or \$5.75 for a package of 5 tickets. If a student requires 40 tickets, how much does she save by buying all of the tickets in packages of 5 rather than buying 40 single tickets?

We will calculate each cost, then compare.

Cost singles:

$C_s = 40 \cdot \$1.50$

$= \$60$

Cost packages:

$C_p = 40 \cdot \left(\frac{\$5.75}{5}\right)$

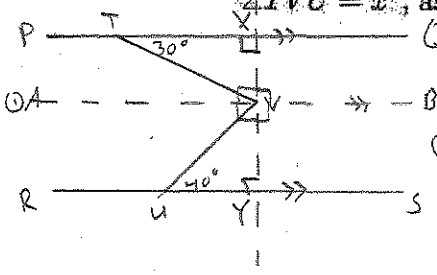
$= \$46$

it is \$60 to buy all singles and \$46 to buy all packages.

$\$60 - \$46 = \$14$

She saves \$14 by purchasing packages of 5.

4. Line segments PQ and RS are parallel. Points T, U, V are placed so that $\angle QTV = 30°$, $\angle SUV = 40°$, and $\angle TVU = x°$, as shown. What is the value of x?



① add a third parallel line through V. Call it AB

② add a perpendicular line through V it intersects PQ at X and RS at Y.

③ So $\angle UVY = 90°$, $\angle TXV = 90°$

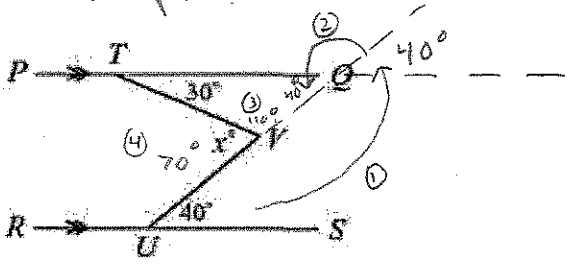
④ now $\angle TVX = 60°$, $\angle UVY = 50°$ by $\angle$s in $\triangle$ sum 180°

⑤ now $\angle AVT = 30°$, $\angle AVU = 40°$ because $\angle AVY$, $\angle AVX = 90°$

⑥ add $\angle AVT + \angle AVU$ to get $x = 70°$

Method 1: extend with auxillary lines.

① the intersection at Q is the same as at U, so we can place 40° there



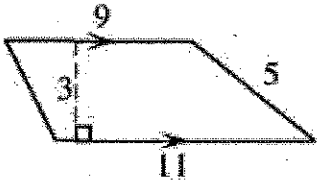
② $\angle$s across intersections are =

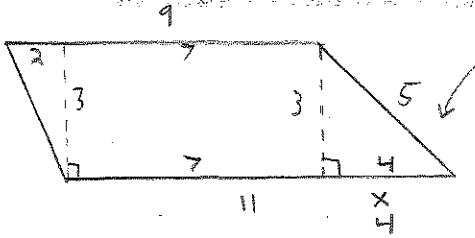
③ $\angle$s in $\triangle$ sum 180°

④ $\angle$s on — sum 180°

$\therefore x = 70°$

5. What is the area of the figure shown?





$3^2 + x^2 = 5^2$

$x^2 = 25 - 9$

$x^2 = 16$

$x = 4$

so the central part of our shape has length 7, which we will add to the diagram and see that we get the upper left portion = 2.

$\nabla = \frac{2 \cdot 3}{2}$ ▭ $= 3 \cdot 7$ ◺ $= \frac{3 \cdot 4}{2}$

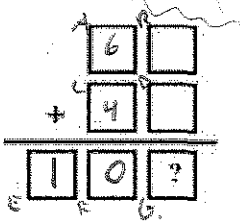
$3 + 21 + 6 = 30$

The total area is 30.

OR...

$A = \left(\frac{b_1 + b_2}{2}\right) \cdot h$

6. In the addition of two 2-digit numbers, each blank space, including those in the answer, is to be filled with one of the digits 0, 1, 2, 3, 4, 5, 6, each used exactly once. The units digit of the sum is



We can only use each one time. Label the spots as indicated

① E must be 1 because the sum of two 2-digit #s can't exceed 198.

② The biggest options for AB and CD are 64 + 53 or 63 + 54. Each of these yield EFG < 120, so F is at most 1. Since 1 is taken, F = 0.

③ Now, examine B + D. B + D can't be at most 11 (5+6) but the G = 1 (which it can't)

B + D cannot be 10 since then G = 0 (which it can't)

so B + D ≤ 9.

Therefore, A + C = 10, so A, C = 6, 4 (only possibilities)

WLOG, let A = 6, C = 4.

④ Now we only have 2, 3, 5 remaining. 2 + 5 ≠ 3, 3 + 5 ≠ 2,

2 + 3 = 5 so we have B and D = 2 and 3 and

G = 5